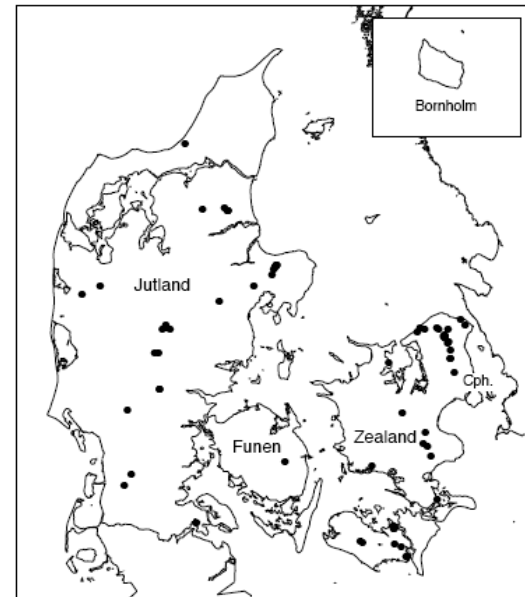


Mathematical models of height growth in Lake States (USA) and Denmark.

R. A. Leary

Example application of Gowin Vee with triangle ontology + Lakehead framework.



1. Central event or system or process

Conceptual

Methodological

5. System component properties, concepts

2. Statements / Questions to focus research

9. Experimental/sampling design

6. Scientific hypotheses/propositions

10. Measurements/data

3. Literature/ references

11. Mathematical analysis of data

7. Deductions from sci. hypo/ Scientific inference

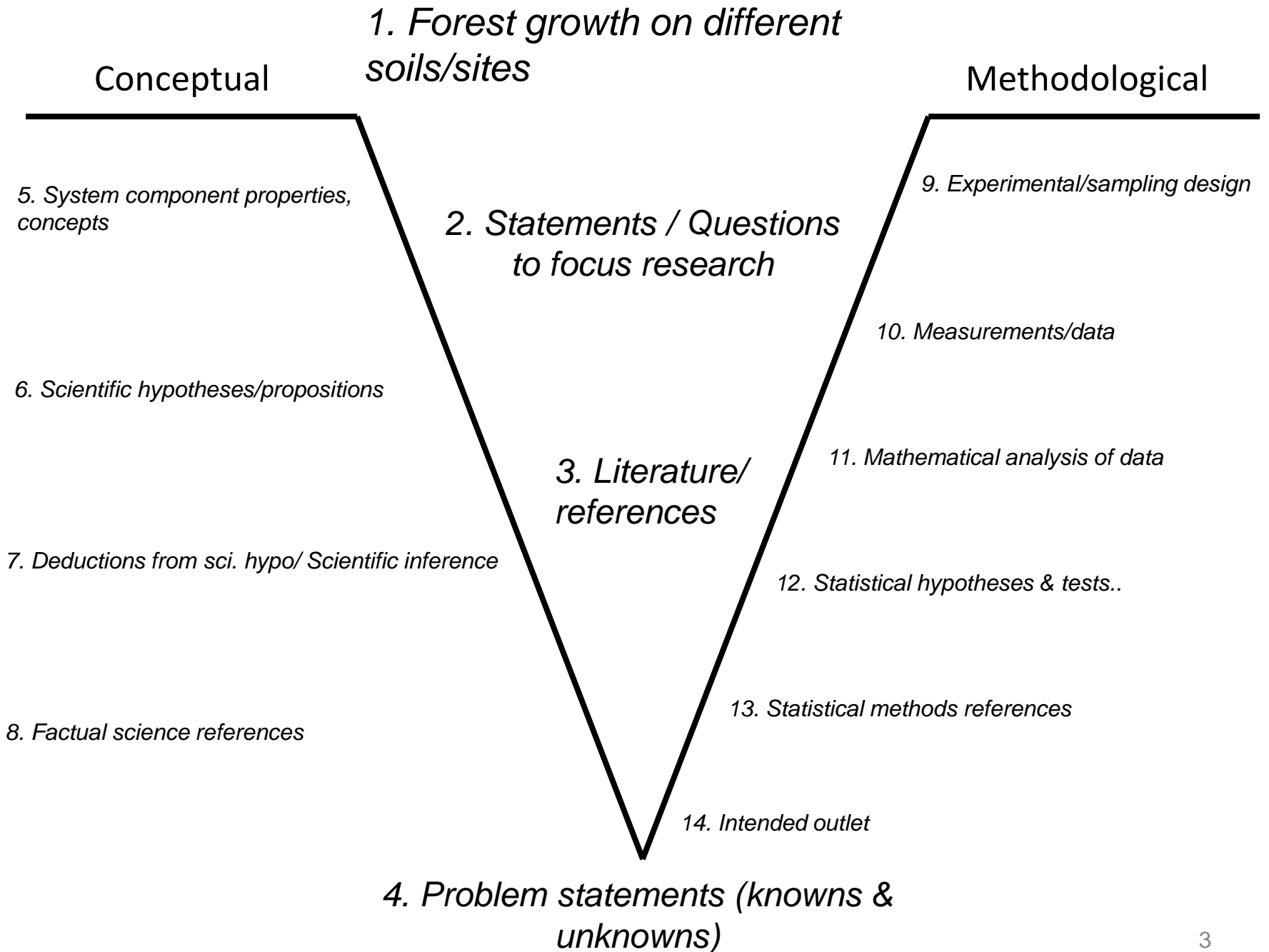
12. Statistical hypotheses & tests..

8. Factual science references

13. Statistical methods references

14. Intended outlet

4. Problem statements (knowns & unknowns)



1. Forest growth on different soils/sites

Conceptual

Methodological

2. Forest systems require moisture, nutrients, heat and light for growth. Soils supply moisture and nutrients, and vary considerably more than heat and light.

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4b. Are there primary soil factors responsible for the observed differences?

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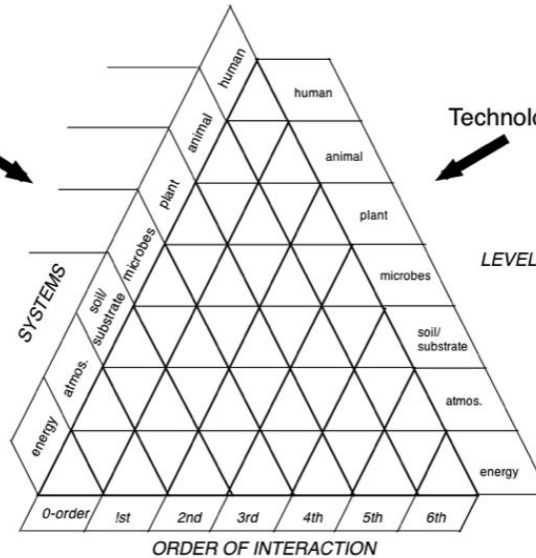
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Science

Technology



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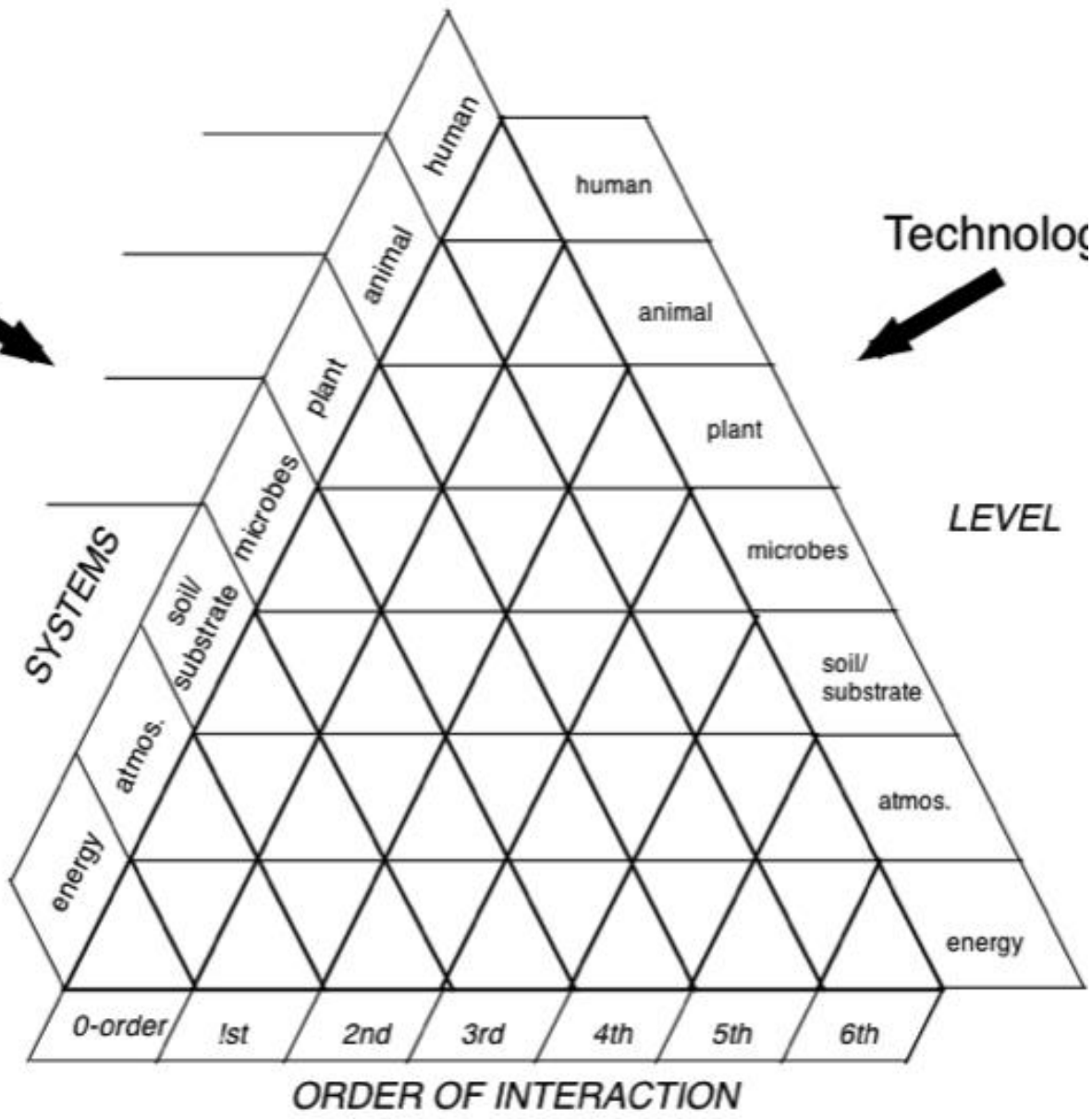
14. Intended outlet

4b. Are there primary soil factors responsible for the observed differences?

Science



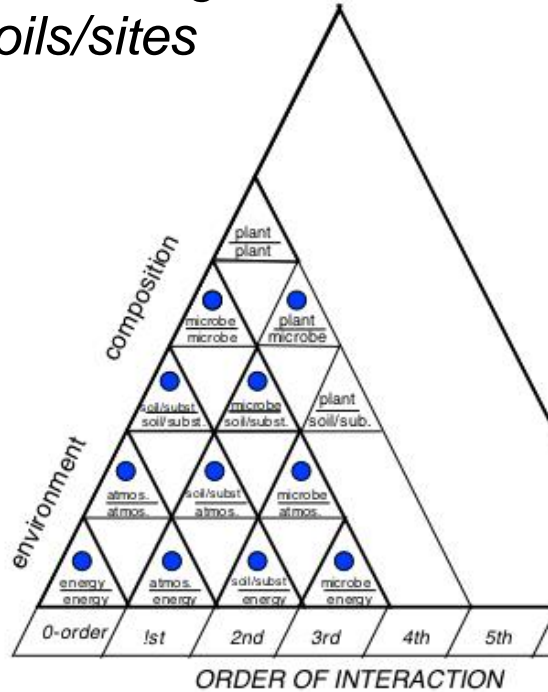
Technology



1. Forest growth on different soils/sites

Conceptual

Methodological



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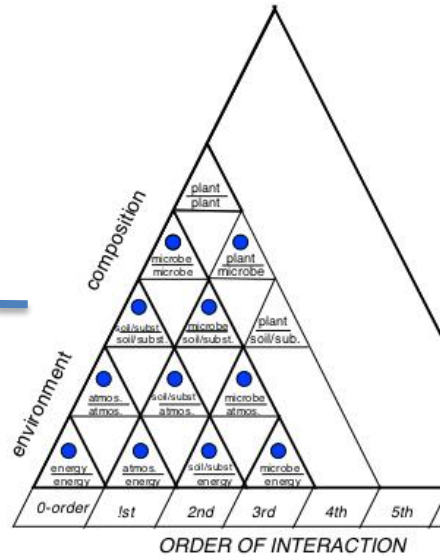
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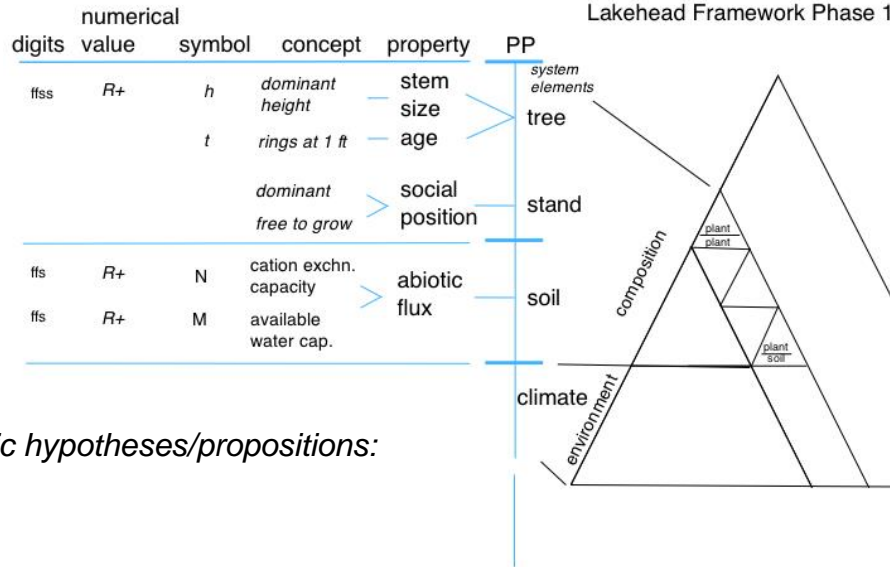
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symbol	concept	property	PP
h	dominant height	stem size	tree
t	age at 1 ft	age	
	dominant tree to grow	social position	stand
N	cation exchange cap.	abiotic flux	soil
M	available soil moisture		
			environment

6. Scientific hypotheses/propositions:

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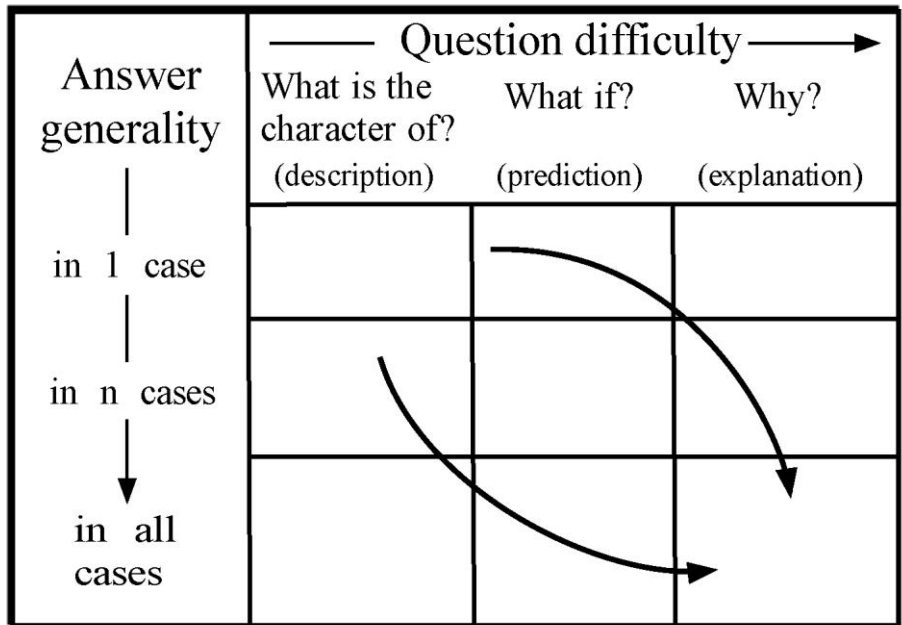
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4b. Are there primary soil factors responsible for the observed



**Model
performance
criteria**
(M.B. I: pg 321)

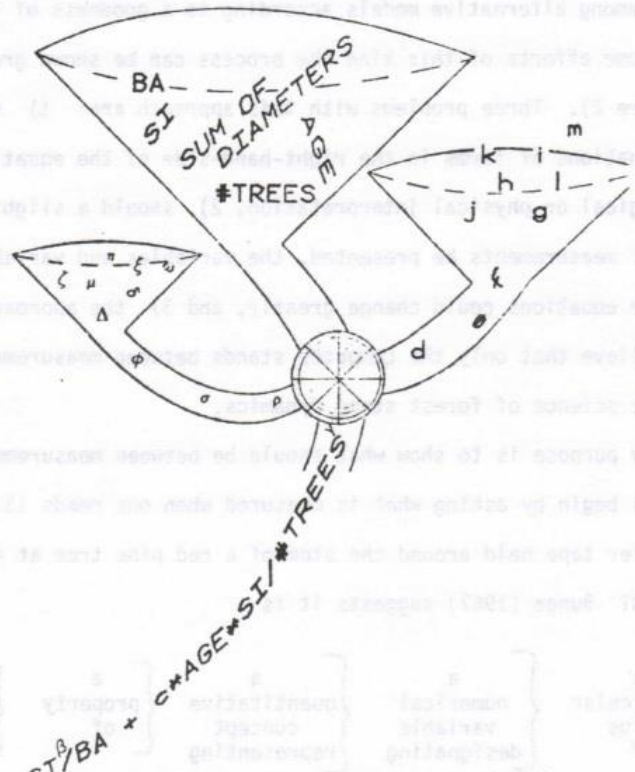
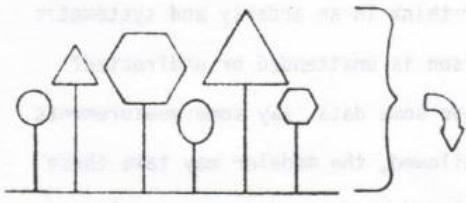
goodness of
fit



interpretation
of model
constants



theorification
potential



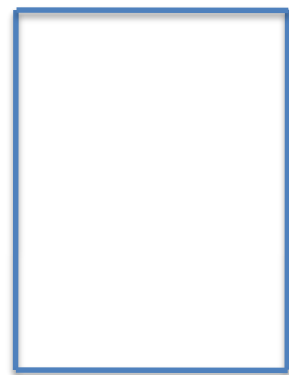
$$\Delta Y = a * BA^a + b * SI^b / BA + c * AGE * SI / \# TREES$$

Figure 2. The goodness of fit fixation of some modelers produces some interesting and temporarily useful results. But try interpreting terms in their right-hand-sides.

A design for survivor growth models: pp 62-81 in Proceedings of a Workshop at Lakehead University: Forecasting forest stand dynamics. 1980. K. M. Brown and F. R. Clarke (eds)



The elusive formula of best fit.
A comprehensive new machine program.
1958 – USDA Forest Service



All possible regressions with less computation. 1971

1. Forest growth on different soils/sites

Conceptual

Methodological

5. System component properties, concepts

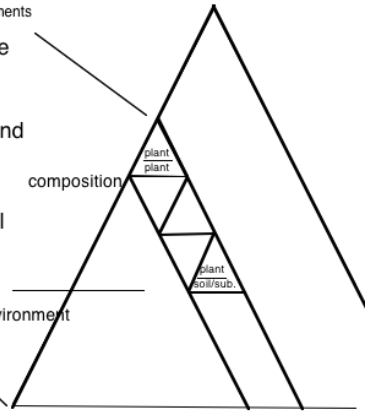
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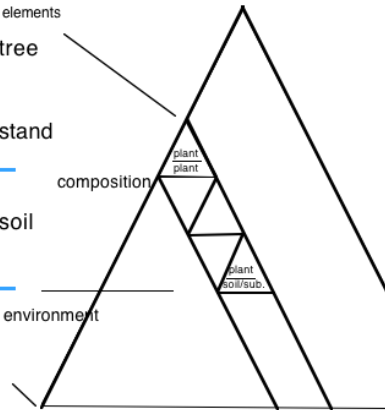
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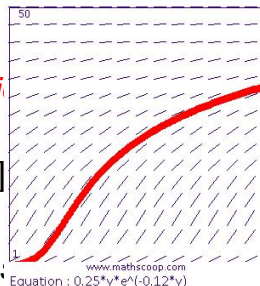
3. Literature references

12. Statistical hypotheses & tests..

7. Deductions from sci. hypo/ Scientific inference

slope fi

$$dh / dt = a h \text{Exp}[-b h]$$



13. Statistical methods references

14. Intended outlet

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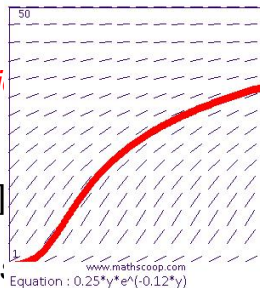
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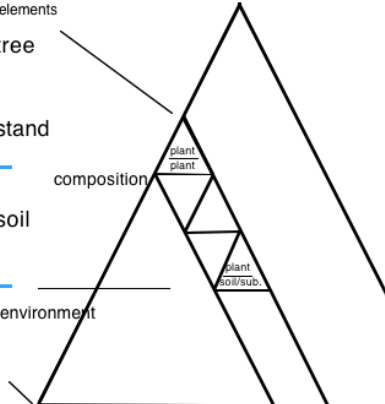
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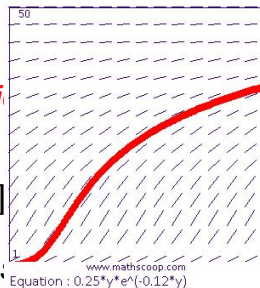
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slope fi

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8. Factual science reference:

Equation : $0.25^y * e^{(-0.12^*y)}$

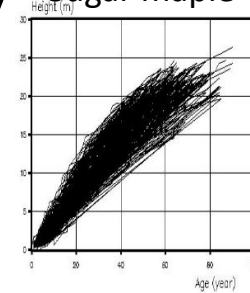
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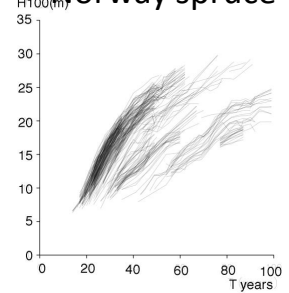
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Sugar maple



Norway spruce



11. Mathematical analysis of data

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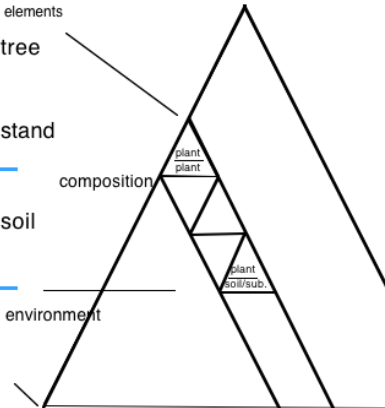
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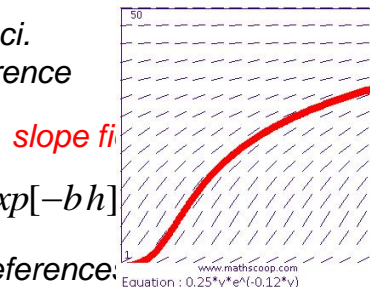


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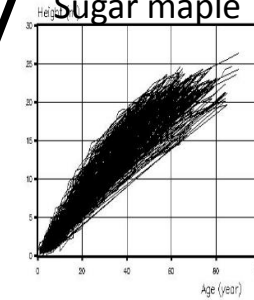
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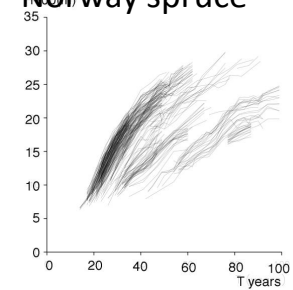
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Norway spruce



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$h_{i,j}$ = top height of j th plot at i th time

\hat{a}_j = specific parameter for j th plot

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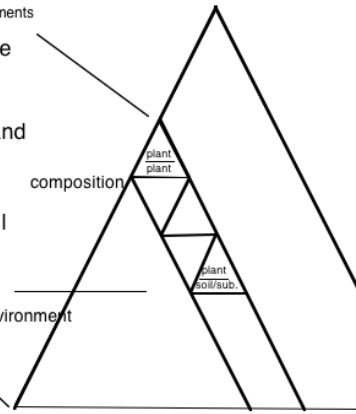
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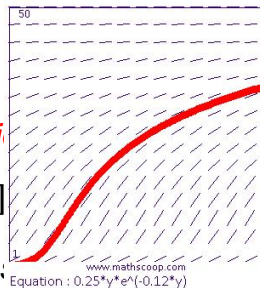
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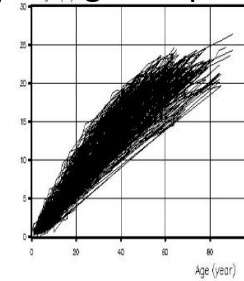
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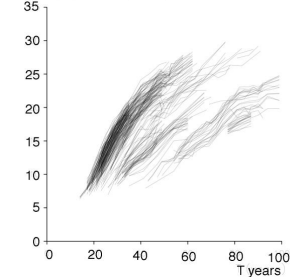
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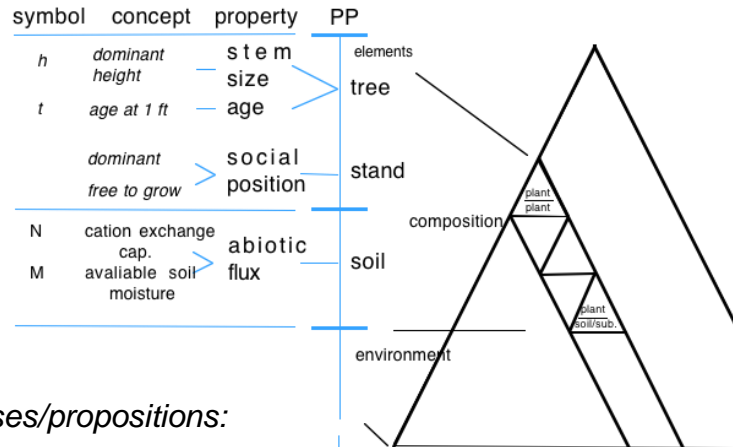
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Conceptual

Methodological

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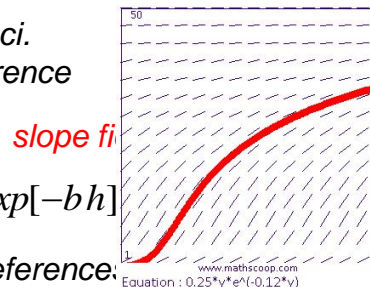


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<http://www.mathscoop.com>

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\hat{b} = common parameter for all plots

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$$\hat{a}_j = f(\text{soil, site properties})$$

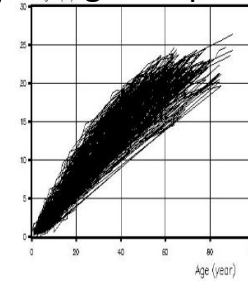
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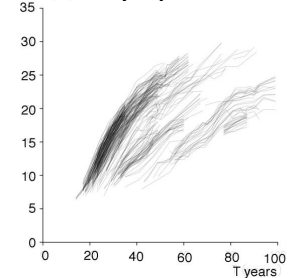
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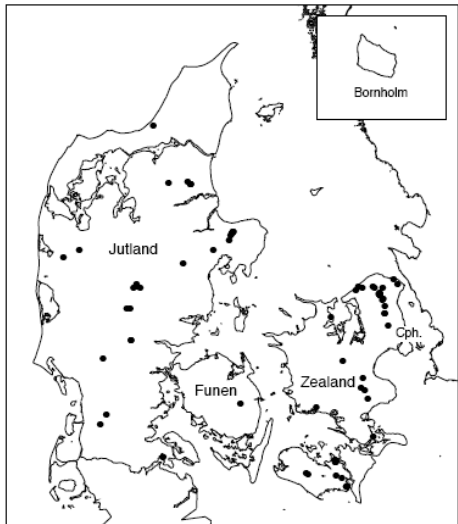
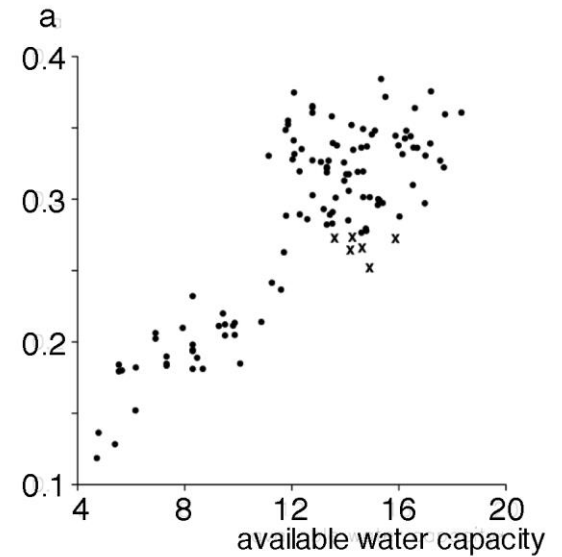
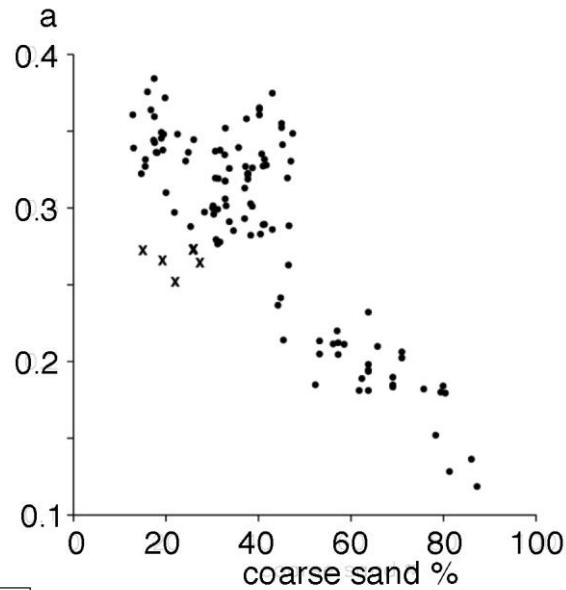
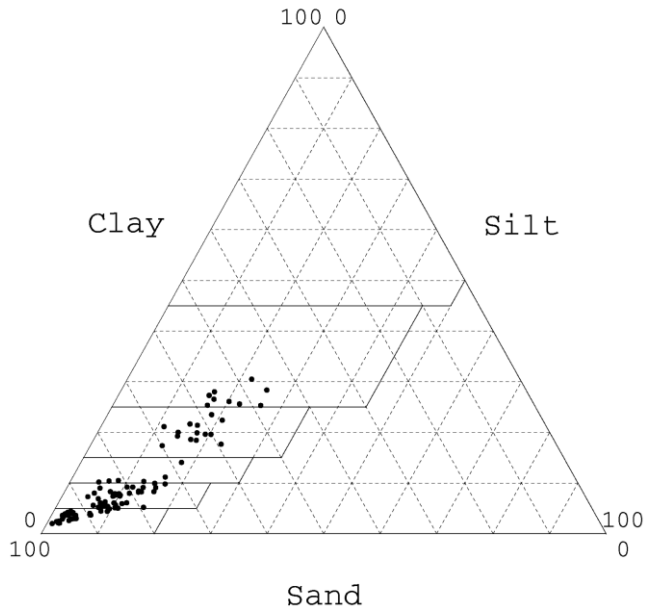
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4b. Are there primary soil factors responsible for the observed differences?

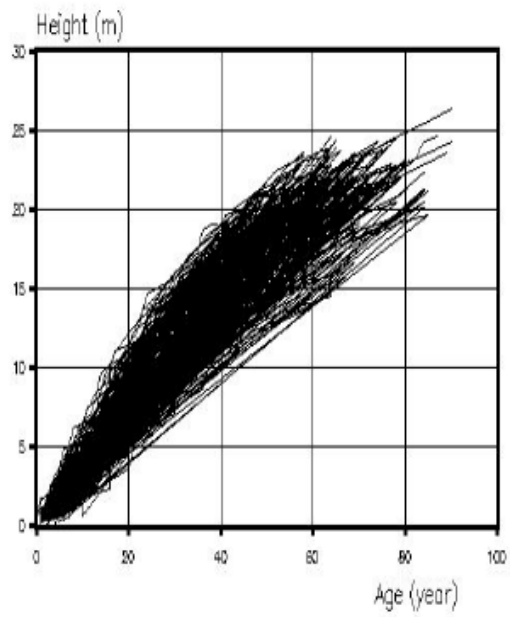


European standards:

- coarse sand (0.2-2.0 mm),
- fine sand (0.02-0.2 mm),
- silt (0.002-0.02),
- clay (<0.002 mm).



Running - Rich Diesslin © 1984,2003
Drawing for Animation





USA sand is just plain sand,
doesn't matter how fine or coarse!!

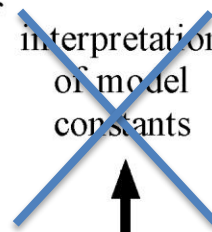
Answer generality	Question difficulty →		
	What is the character of? (description)	What if? (prediction)	Why? (explanation)
in 1 case			
in n cases			
in all cases			

**Model
performance
criteria**
(M.B. I: pg 321)

goodness of
fit

~~interpretation
of model
constants~~

theorification
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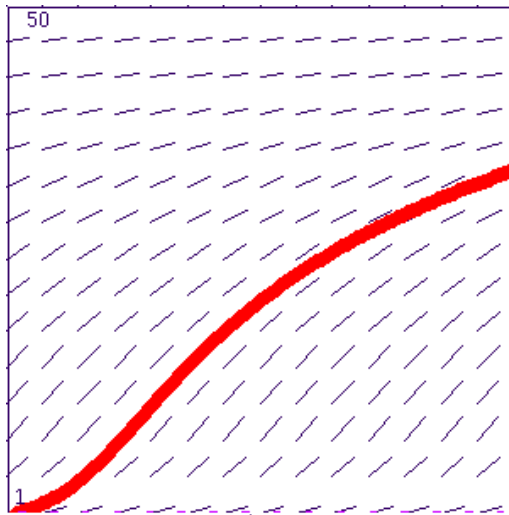


Thank you

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clay (<0.002 mm).

Madsen and Platou (1983) developed a model for available water capacity for Danish conditions. Their model is:

$$\text{AWC (\%)} = 1.96 * \text{organic matter \%} + 0.02 * \text{clay \%} + 0.34 * \text{silt \%} + 0.17 * \text{fine sand \%} + 2.26$$



Equation : $0.15 * y * e^{(-0.10 * y)}$

- | | |
|-----------------|---------------------|
| 1) (2.00, 1.00) | 7) (8.00, 2.09) |
| 2) (3.00, 1.14) | 8) (9.00, 2.34) |
| 3) (4.00, 1.29) | 9) (10.00, 2.62) |
| 4) (5.00, 1.46) | 10) (11.00, 2.92) |
| 5) (6.00, 1.65) | 11) (12.00, 3.25) |
| 6) (7.00, 1.86) | ... |
| | 99) (100.00, 34.05) |

