Experimental Design

IUFRO-SPDC

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For Three Men
The Civil War
Wasn't Hell.
It Was
Practice!
CLINT EASTWOOD
"THE GOOD,
THE BAD AND
THE UGLY"
LEE VAN CLEEF
ALDO GIUFFRE
MARIO BREGA
ELI WALLACH
SERGIO LEONE
TECHNISCOPE TECHNICOLOR

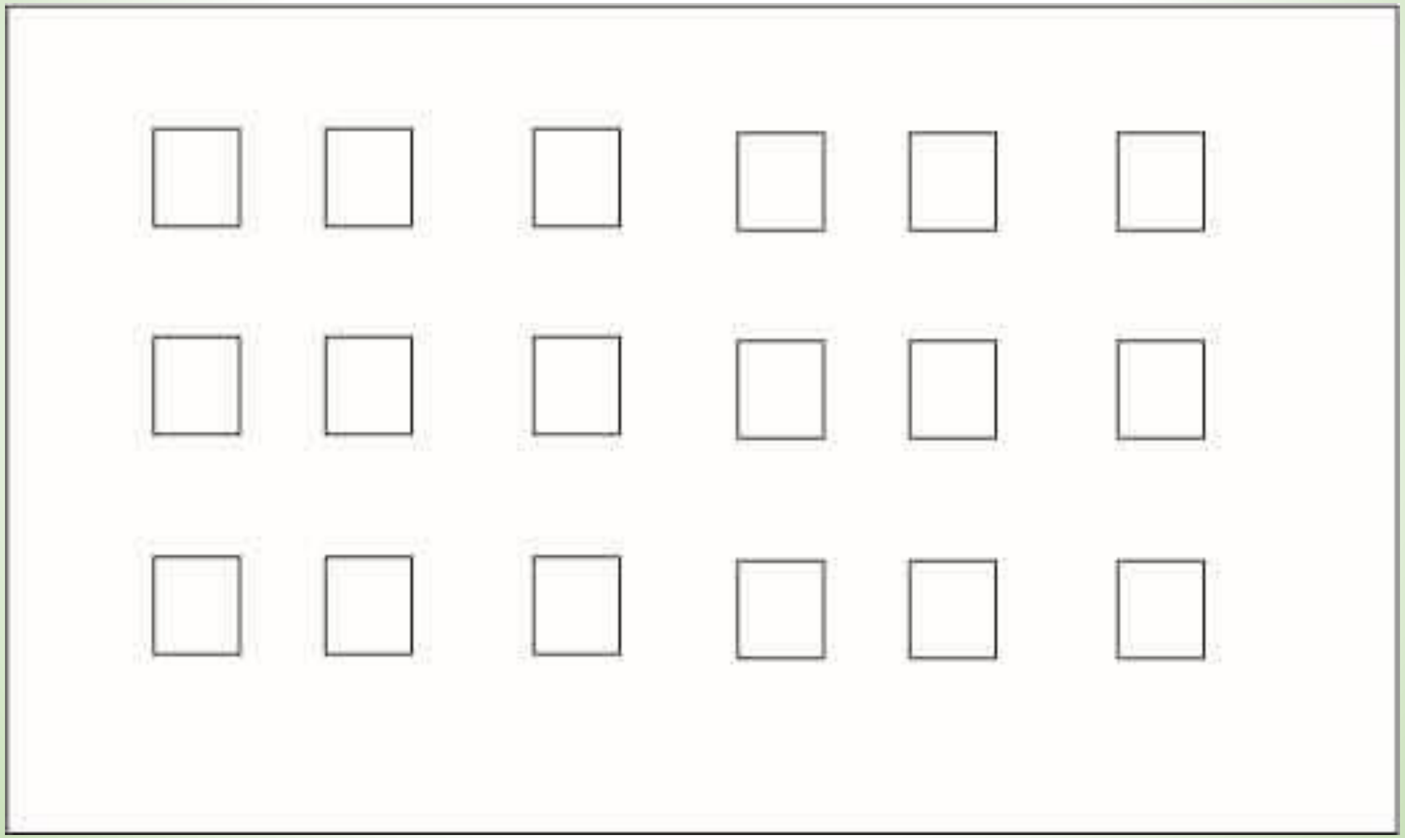
Three scenarios

- The Good
  - You designed the experiment
  - You have the data
  - Now what?
- The Bad
  - Someone else designed the experiment
  - They explain how they laid out the treatments
  - You get the data
  - Now what?
- The Ugly
  - Your boss hands you a file
  - In the file is a map, a brief description of an experiment
  - and…………………… the data sheets
  - Now what?

Experimental Design

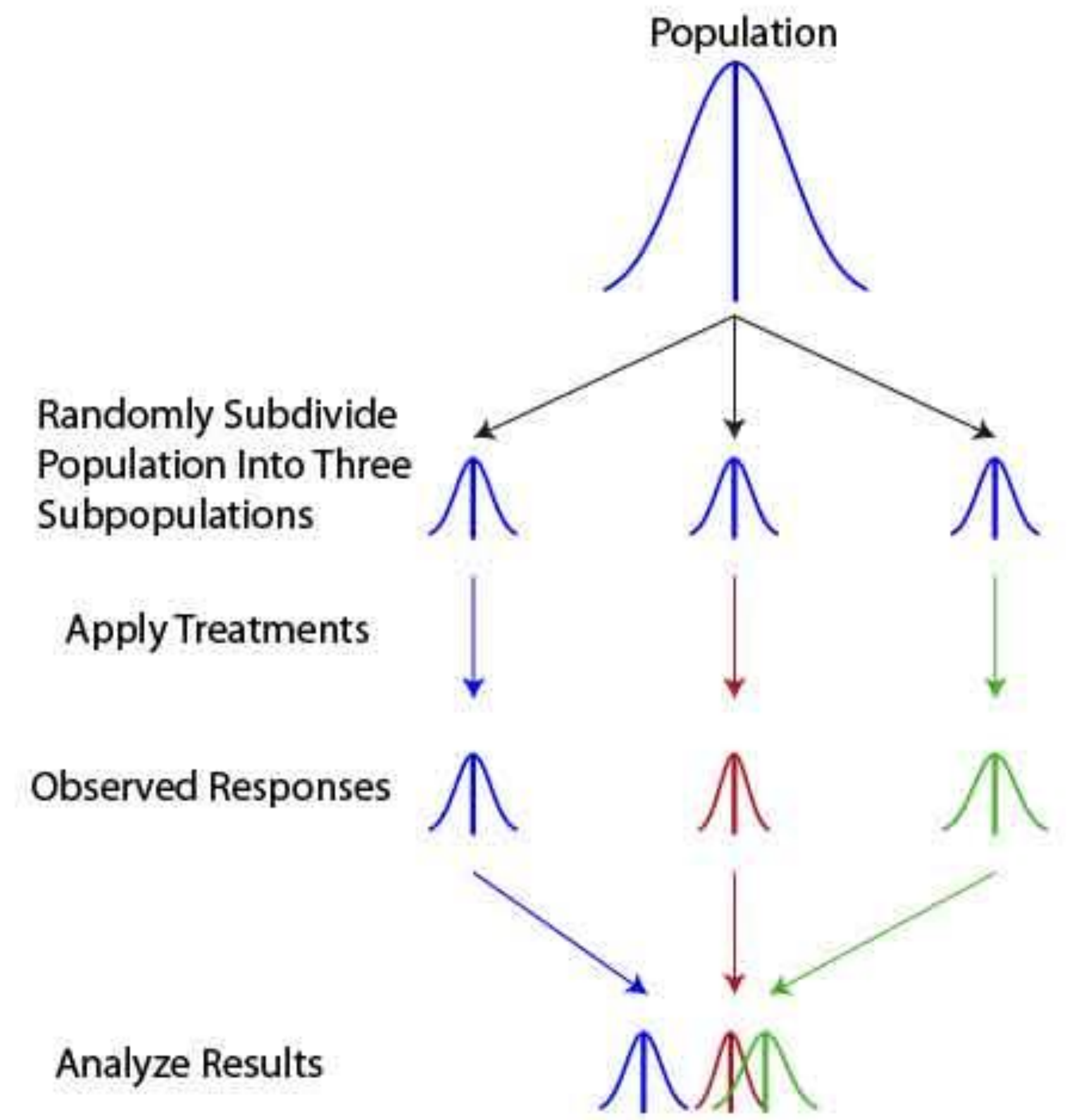
- Concerned with the analysis of data
- “Significant” effects are determined by comparing within group means and variation to between group variation
- In designing experiments, we attempt to minimize within group variation and maximize between group variation

Our Experimental Population



Our Experiment

- Treatment A is hypothesized to effect the response variable that is of interest
- 18 Experimental Units
- 3 Treatment levels:
    - A1 = 0 (Control)
    - A2 = X
    - A3 = 2X
- 6 Replicates


Population
Randomly Subdivide Population Into Three Subpopulations
Apply Treatments
Observed Responses
Analyze Results

Logic of Analysis of Variance

- We start with a uniform population
- Randomly divide it into subpopulations
- Apply a treatment that we expect to influence the subpopulations' means
- We measure effect by examining variation within each treatment to variation between each treatment
- If treatment has "No Effect" then the three means will have the same mean as the original population and the between treatment variation will equal 0
- So, if Treatment variation is small relative to Population variation, then there is no effect
- Conversely, if Treatment variation is large (ie, big differences between treatment means) relative to Population variation, then there is an effect
- Thus we test differences in means by assessing proportions of variation

Statistical Hypotheses

- Null Hypothesis
  - There is no differences between all of the means
  - $\mu 1 = \mu 2 = \mu 3$
- Alternative Hypothesis
  - At least one mean is different
  - $\mu 1 \neq \mu 2 = \mu 3$
  - $\mu 1 = \mu 2 \neq \mu 3$
  - $\mu 1 \neq \mu 2 \neq \mu 3$
- Specific Hypothesis
- General Hypothesis

Statistical Test

- We construct our "Test" statistic assuming the Null hypothesis is true
- If the Null hypothesis is true, the test statistic should be 0 (no difference)
- Most likely (and hopefully) our test statistic will be >> 0
- Because we have a sample we have sampling error
- We learned yesterday that the sampling error causes differences in our estimate of the mean

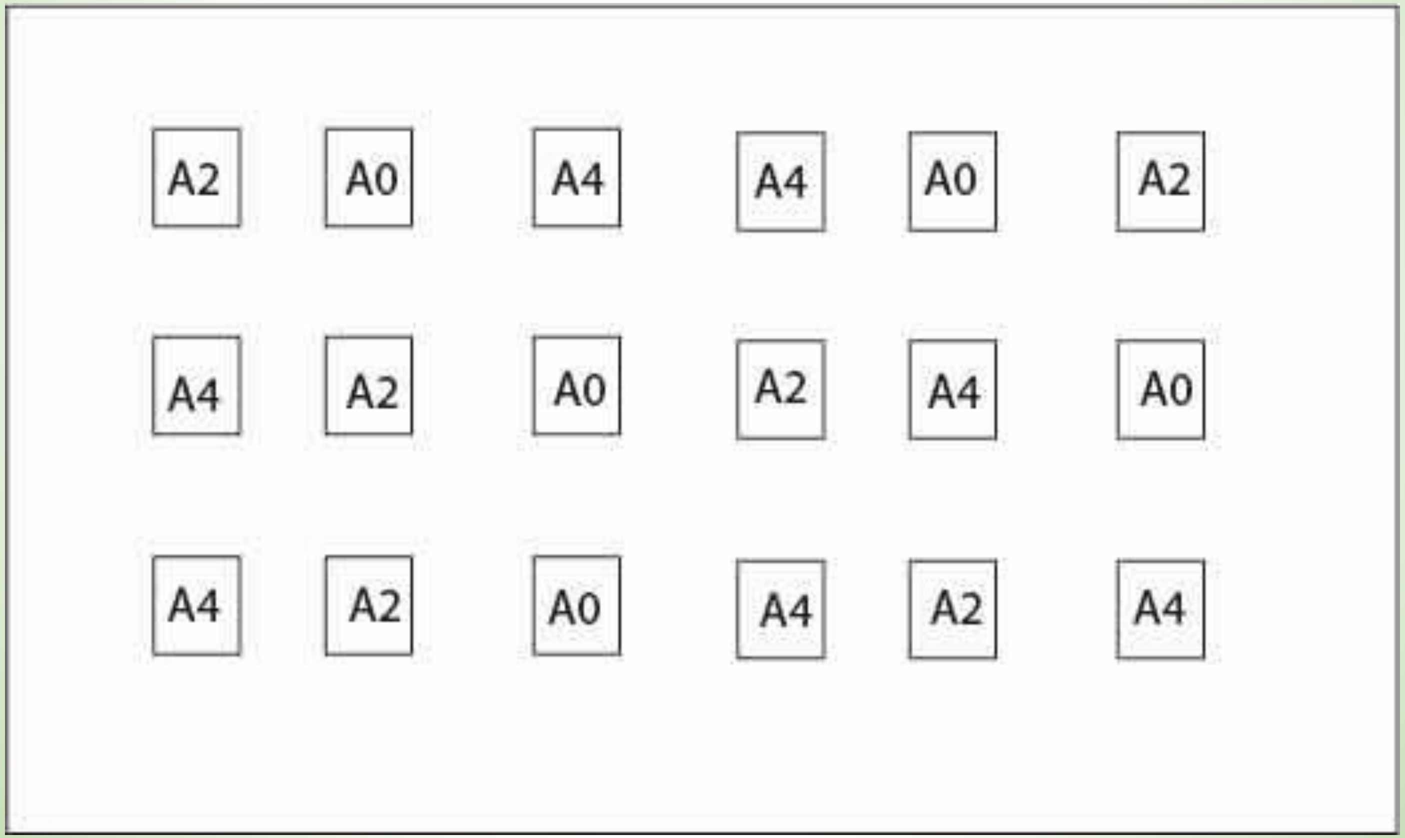
Statistical Test

- So we could obtain a test statistic >> 0, because of sampling error
- Therefore, we assess, given the variability in our population, what is the probability that a difference as large as we have observed, is due to sampling error
- If that probability is small, then we assume the difference is not due to sample error, but due to our treatment, and we conclude that we have significant treatment effects

Experiment 1 – Simple One Way ANOVA

- 18 Experimental Units
- One treatment (A)
- Three treatment levels
  - A1 = 0 (Control)
  - A2 = X
  - A3 = 2X
- 6 Replicates
- Treatment level is randomly assigned to each experimental unit

Experiment 1 – Layout Map



The ANOVA Table

Source	DF	Sum of Squares	Mean Squares	F-statistic	P(F)
Between (treatment)	k-1	$r\sum_{i=1}^{k}\left(\overline{Y}_{i.}-\overline{Y}_{..}\right)^2$	SST/(k-1)	MST/MSR	
Within (residual)	k(r-1)	$\sum_{i=1}^{k}\sum_{j=1}^{r}\left(Y_{ij}-\overline{Y}_{i.}\right)^2$	SSR/(k(r-1))		
Total	kr-1	$\sum_{i=1}^{k}\sum_{j=1}^{r}\left(Y_{ij}-\overline{Y}_{..}\right)^2$			

The ANOVA Linear Model

- $Y(ij) = \mu + T(i) + e(ij)$
- The model implies expected mean squares
- If you can determine the expected mean squares, you can analyze any experimental design
- Fortunately there are a few “rules” that make this job relatively easy

Expected Mean Squares – One way ANOVA

- Write the variable terms in the model as row headings, include subscripts, bracket subscripts for nested factors

Variables				
Ai				
ej(i)				

Expected Mean Squares – One way ANOVA

- Write the subscripts in the model as column headings; over each subscript write F if the factor levels are fixed, R if they are random. Also write the number of observations

	3	6		
	F	R		
	i	j		EMS
Variables				
Ai				
ej(i)				

Expected Mean Squares – One way ANOVA

- For each row (each term in the model) copy the number of observations under each subscript, providing the subscript does not appear in the row heading

	3	6		
	F	R		
	i	j		EMS
Variables				
Ai		6		
ej(i)				

Expected Mean Squares – One way ANOVA

- For any bracketed subscripts in the model, place a 1 under those subscripts that are in the brackets

	3	6		
	F	R		
	i	j		EMS
Variables				
Ai		6		
ej(i)	1			

Expected Mean Squares – One way ANOVA

- Fill the remaining cells with 0 or 1, depending upon whether the factor is F (0) or R (1)

	3	6		
	F	R		
	i	j		EMS
Variables				
Ai	0	6		
ej(i)	1	1		

Expected Mean Squares – One way ANOVA

- Expected mean squares is found by covering the column(s) that contain non-bracketed subscript letters; multiply the remaining numbers in each row, these products are the coefficients for the factor contribution to EMS

	[redacted]	6		
		R		
		j		EMS
Variables				
Ai		6		6ø(T) + σ^2(e)
ej(i)		1		

Expected Mean Squares – One way ANOVA

- Expected mean squares is found by covering the column(s) that contain non-bracketed subscript letters; multiply the remaining numbers in each row, these products are the coefficients for the factor contribution to EMS

	3	[redacted]		
	F	[redacted]		
	i	[redacted]		EMS
Variables		[redacted]		
Ai	0	[redacted]		6ø(T) + σ^2(e)
ej(i)	1	[redacted]		σ^2(e)

More complicated designs

- Two-way ANOVA fixed factors
- Two-way ANOVA fixed and random factors
- Randomized Block Design
- Nested Design

Experiment 2: Two-way Anova with Fixed Factors

- 18 experimental units
- Treatment A
  - A0
  - A2
  - A4
- Treatment B
  - B0
  - B1
- 6 Treatment combinations
- 3 Replicates
- Linear model: $Y = \mu + ø(A) + ø(B) + ø(AB) + e$

Expected Mean Squares – two way ANOVA

- Factors listed

	3	2	3	
	F	F	R	
Variables	i	j	k	EMS
Ai				
Bj				
Abij				
ek(ij)				

Expected Mean Squares – two way ANOVA

- For each row (each term in the model) copy the number of observations under each subscript, providing the subscript does not appear in the row heading

	3	2	3	
	F	F	R	
Variables	i	j	k	EMS
Ai		2	3	
Bj	3		3	
Abij				
ek(ij)				

Expected Mean Squares – two way ANOVA

- For any bracketed subscripts in the model, place a 1 under those subscripts that are in the brackets

	3	2	3	
	F	F	R	
Variables	i	j	k	EMS
Ai		2	3	
Bj	3		3	
Abij				
ek(ij)	1	1		

Expected Mean Squares – two way ANOVA

- Fill the remaining cells with 0 or 1, depending upon whether the factor is F (0) or R (1)

	3	2	3	
	F	F	R	
Variables	i	j	k	EMS
Ai	0	2	3	
Bj	3	0	3	
Abij	0	0	3	
ek(ij)	1	1	1	

Expected Mean Squares – two way ANOVA

- Expected mean squares is found by covering the column(s) that contain non-bracketed subscript letters; multiply the remaining numbers in each row, these products are the coefficients for the factor contribution to EMS

	[redacted]	2	3	
	[redacted]	F	R	
Variables	[redacted]	j	k	EMS
Ai	[redacted]	2	3	6ø(A) + 0ø(B) + 0ø(AB)+ σ^2(e)
Bj	[redacted]	0	3	
Abij	[redacted]	0	3	
ek(ij)	[redacted]	1	1	

Expected Mean Squares – two way ANOVA

- Expected mean squares is found by covering the column(s) that contain non-bracketed subscript letters; multiply the remaining numbers in each row, these products are the coefficients for the factor contribution to EMS

	3	[redacted]	3	
	F	[redacted]	R	
Variables	i	[redacted]	k	EMS
Ai	0	[redacted]	3	6ø(A) + 0ø(B) + 0ø(AB)+ σ^2(e)
Bj	3	[redacted]	3	0ø(A) + 9ø(B) + 0ø(AB)+ σ^2(e)
Abij	0	[redacted]	3	
ek(ij)	1	[redacted]	1	

Expected Mean Squares – two way ANOVA

- Expected mean squares is found by covering the column(s) that contain non-bracketed subscript letters; multiply the remaining numbers in each row, these products are the coefficients for the factor contribution to EMS

	[redacted]	3	
		R	
Variables		k	EMS
Ai		3	6ø(A) + 0ø(B) + 0ø(AB)+ σ^2(e)
Bj		3	0ø(A) + 9ø(B) + 0ø(AB)+ σ^2(e)
Abij		3	0ø(A) + 0ø(B) + 3ø(AB)+ σ^2(e)
ek(ij)		1	

Expected Mean Squares – two way ANOVA

- Expected mean squares is found by covering the column(s) that contain non-bracketed subscript letters; multiply the remaining numbers in each row, these products are the coefficients for the factor contribution to EMS

	3	2	[redacted]	
	F	F		
Variables	i	j		EMS
Ai	0	2		6ø(A) + 0ø(B) + 0ø(AB)+ σ^2(e)
Bj	3	0		0ø(A) + 9ø(B) + 0ø(AB)+ σ^2(e)
Abij	0	0		3ø(AB)+ σ^2(e)
ek(ij)	1	1		σ^2(e)

Experiment 3: Two-way ANOVA with A fixed and B random

- Same design as last time
- B is a nuisance factor that we cannot control, but only observe it level (ie, we have a "random" sample of levels of B)
- Linear model: $Y = \mu + ø(A) + ø(B) + ø(AB) + e$

Expected Mean Squares – Experiment 3

- For each row (each term in the model) copy the number of observations under each subscript, providing the subscript does not appear in the row heading

	3	2	3	
	F	R	R	
Variables	i	j	k	EMS
Ai		2	3	
Bj	3		3	
Abij			3	
ek(ij)				

Expected Mean Squares – Experiment 3

- For any bracketed subscripts in the model, place a 1 under those subscripts that are in the brackets

	3	2	3	
	F	R	R	
Variables	i	j	k	EMS
Ai		2	3	
Bj	3		3	
Abij			3	
ek(ij)	1	1		

Expected Mean Squares – Experiment 3

- Fill the remaining cells with 0 or 1, depending upon whether the factor is F (0) or R (1)

	3	2	3	
	F	R	R	
Variables	i	j	k	EMS
Ai	0	2	3	
Bj	3	1	3	
Abij	0	1	3	
ek(ij)	1	1	1	

Expected Mean Squares – Experiment 3

- Expected mean squares is found by covering the column(s) that contain non-bracketed subscript letters; multiply the remaining numbers in each row, these products are the coefficients for the factor contribution to EMS

	[redacted]	2	3	
	[redacted]	R	R	
Variables	[redacted]	j	k	EMS
Ai	[redacted]	2	3	6ø(A) + 6ø(AB) + σ^2(e)
Bj	[redacted]	1	3	
Abij	[redacted]	1	3	
ek(ij)	[redacted]	1	1	

Expected Mean Squares – Experiment 3

- Expected mean squares is found by covering the column(s) that contain non-bracketed subscript letters; multiply the remaining numbers in each row, these products are the coefficients for the factor contribution to EMS

	3	[redacted]	3	
	F	[redacted]	R	
Variables	i	[redacted]	k	EMS
Ai	0	[redacted]	3	6ø(A) + 6ø(AB) + σ^2(e)
Bj	3	[redacted]	3	9ø(B) + 0ø(AB) + σ^2(e)
Abij	0	[redacted]	3	
ek(ij)	1	[redacted]	1	

Expected Mean Squares – Experiment 3

- Expected mean squares is found by covering the column(s) that contain non-bracketed subscript letters; multiply the remaining numbers in each row, these products are the coefficients for the factor contribution to EMS

	[redacted]	3	
	[redacted]	R	
Variables	[redacted]	k	EMS
Ai	[redacted]	3	6ø(A) + 6ø(AB) + σ^2(e)
Bj	[redacted]	3	9ø(B) + 0ø(AB) + σ^2(e)
Abij	[redacted]	3	3ø(AB) + σ^2(e)
ek(ij)	[redacted]	1	

Expected Mean Squares – Experiment 3

- Expected mean squares is found by covering the column(s) that contain non-bracketed subscript letters; multiply the remaining numbers in each row, these products are the coefficients for the factor contribution to EMS

	3	2	[redacted]	
	F	R	[redacted]	
Variables	i	j	[redacted]	EMS
Ai	0	2	[redacted]	6ø(A) + 6ø(AB) + σ^2(e)
Bj	3	1	[redacted]	9ø(B) + 0ø(AB) + σ^2(e)
Abij	0	1	[redacted]	3ø(AB) + σ^2(e)
ek(ij)	1	1	[redacted]	σ^2(e)